Compliance and Pure Erotetic Implication

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Compliance and Pure Erotetic Implication

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Abstract: Notions of compliance and pure erotetic implication are introduced and compared in the context of grasping the dependency between questions.

Keywords: question dependency, erotetic implication, compliance, IEL, inquisitive semantics

1 Introduction

In this paper I will be addressing a situation where a question is given as a response to a question. I will present how this type of reply to a question—which I will call a *question response*—is grasped with the notion of compliance (developed within inquisitive semantics—INQ, cf. (Groenendijk, 2009; Groenendijk & Roelofsen, 2011)) and by the notion of erotetic implication (which comes from Inferential Erotetic Logic—IEL framework, cf. (Wiśniewski, 1995, 2013)).

Łupkowski and Ginzburg (2013) describe a typology of question responses based on the British National Corpus (BNC) study. Among these various question responses one type is especially interesting, namely *dependent question* responses. The rationale behind dependent questions can be summarised as follows (Ginzburg, 2012, p. 57): if Q depends on Q_1 then discussions of Q_1 will necessarily bring about the provision of information about Q.

Definition 1 (Ginzburg, 2012, p. 57) Q depends on Q_1 *iff any proposition* p *such that* p *resolves* Q_1*, also satisfies* p *entails* r *such that* r *is about* Q*. This allows to say that* Q_1 *can be used to respond to* Q *if* Q depends on Q_1.

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In other words, Q_1 is an acceptable response to Q. One may easily notice that this type of question response captures the intuition of introducing a sub-question which will somehow help to retrieve the answer to the initial question. The following example illustrate this idea:

A: Do you want me to <*pause*> push it round?
B: Is it really disturbing you? [FM1, 679–680][2]
(i.e. *Whether I want you to push it depends on whether it really disturbs you*)

Let us now take a closer look at how this relation between questions is expressed in INQ and IEL.

2 The notion of compliance

In the framework of inquisitive semantics, the dependency relation is analysed in terms of *compliance*. The intuition behind the notion of compliance is to provide a criterion to 'judge whether a certain conversational move makes a significant contribution to resolving a given issue' (Groenendijk & Roelofsen, 2011, p. 167). If we take two conversational moves: the initiative A and the response B, there are two ways in which B may be compliant with A (cf. Groenendijk & Roelofsen, 2011, p. 168):

(a) B may partially *resolve* the issue raised by A (answerhood).

(b) B may replace the issue raised by A by an easier to answer sub-issue (subquestionhood).[3]

Here I will be interested only in the case when we are dealing with subquestionhood. Before I will be able to give the definition of compliance, first I will introduce (after Wiśniewski and Leszczyńska-Jasion (2013, pp. 6–12)) the necessary concepts of INQ, especially the notion of question used in this framework.

[2]This notation indicates the British National Corpus file (FM1) together with the sentence numbers (679–680).

[3]In the inquisitive semantics also combinations of (a) and (b) are possible, i.e. B may partially resolve the issue raised by A and replace the remaining issue by an easier to answer sub-issue.

2.1 INQ basic concepts

Firstly, let us introduce a language $\mathcal{L}_{\mathcal{P}}$. It is a propositional language over a non-empty set of propositional variables $\mathcal{P}$, where $\mathcal{P}$ is either finite or countably infinite. The primitive logical constants of the language are: $\bot, \vee, \wedge, \rightarrow$. Well-formed formulas (wffs) of $\mathcal{L}_{\mathcal{P}}$ are defined as usual.

The letters A, B, C, D, are metalanguage variables for wffs of $\mathcal{L}_{\mathcal{P}}$, and the letters X, Y are metalanguage variables for sets of wffs of the language. The letter p is used below as a metalanguage variable for propositional variables.

$\mathcal{L}_{\mathcal{P}}$ is associated with the set of suitable possible worlds, $\mathcal{W}_{\mathcal{P}}$, being the *model* of $\mathcal{L}_{\mathcal{P}}$. A possible world is identified with indices (that is valuations of $\mathcal{P}$). $\mathcal{W}_{\mathcal{P}}$ is the set of all indices.

A *state* is a subset of $\mathcal{W}_{\mathcal{P}}$ (states are thus sets of possible worlds). I will use the letters σ, τ, γ, to refer to states.

The most important semantic relation between states and wffs is that of *support*. In the case of INQ support, $\succ$, is defined by:

Definition 2 (Wiśniewski & Leszczyńska-Jasion, 2013, p. 6)
Let $\sigma \subseteq \mathcal{W}_{\mathcal{P}}$.

1. $\sigma \succ \mathrm{p}$ *iff for each* $w \in \sigma$: p *is true in* w,[4]
2. $\sigma \succ \bot$ *iff* $\sigma = \emptyset$,
3. $\sigma \succ (A \wedge B)$ *iff* $\sigma \succ A$ *and* $\sigma \succ B$,
4. $\sigma \succ (A \vee B)$ *iff* $\sigma \succ A$ *or* $\sigma \succ B$,
5. $\sigma \succ (A \rightarrow B)$ *iff for each* $\tau \subseteq \sigma$: *if* $\tau \succ A$ *then* $\tau \succ B$.

For our analysis we will use also the notion of the *truth set* of a wff A (in symbols: $|A|$). It is the set of all the worlds from $\mathcal{W}_{\mathcal{P}}$ in which A is true, where the concept of truth is understood classically.

Now we can introduce the concept of a *possibility* for a wff A. Intuitively it is a maximal state supporting A. This might be expressed as follows:

Definition 3 (Wiśniewski & Leszczyńska-Jasion, 2013, p. 9)
A possibility for wff A *is a state* $\sigma \subseteq \mathcal{W}_{\mathcal{P}}$ *such that* $\sigma \succ A$ *and for each* $w \notin \sigma : \sigma \cup \{w\} \not\succ A$.

[4] "p is true in w" means "the value of p under w equals $\mathbf{1}$".

I will use $\lfloor A \rfloor$ to refer to the set of all possibilities for a wff A.

In INQ we may divide all wffs into assertions and inquisitive wffs. The latter are the most interesting from our point of view, because they raise an issue to be solved. When a wff is inquisitive, the set of possibilities for that formula comprises at least two elements. (When a formula has only one possibility it is called assertion.)

Let us now consider a simple example of an inquisitive formula:

$$(p \vee q) \vee \neg(p \vee q) \tag{1}$$

The set of possibilities for (1) is:

$$\{|p|, |q|, |\neg p| \cap |\neg q|\} \tag{2}$$

and its union is just $\mathcal{W}_{\mathcal{P}}$.

Observe that the language $\mathcal{L}_{\mathcal{P}}$ does not include a separate syntactic category of questions. However, some wffs are regarded as *having the property of being a question*, or $\mathcal{Q}$-property for short.

Definition 4 (Wiśniewski & Leszczyńska-Jasion, 2013, p. 11)
A wff A of $\mathcal{L}_{\mathcal{P}}$ has the $\mathcal{Q}$-property iff $|A| = \mathcal{W}_{\mathcal{P}}$.

Where $\mathcal{W}_{\mathcal{P}}$ stands for the model of $\mathcal{L}_{\mathcal{P}}$, and $|A|$ for the truth set of wff A in $\mathcal{W}_{\mathcal{P}}$. An example of a formula having the $\mathcal{Q}$-property is the formula (1). Hence a wff A is (i.e. has the property of being) a question just in case when A is true in each possible world of $\mathcal{W}_{\mathcal{P}}$, the wffs having the $\mathcal{Q}$-property are just classical tautologies.

2.2 Compliance

Let Q be an *initiative* and Q_1 a *response* to the initiative. We also assume that Q and Q_1 are inquisitive formulae and that they have the $\mathcal{Q}$-property (further on I will just call them questions for simplicity). $\lfloor Q \rfloor$ denotes the set of possibilities for Q.

Definition 5 (cf. Groenendijk, 2009, p. 22) *Let $\lfloor Q \rfloor = \{|A_1|, \ldots, |A_n|\}$ and $\lfloor Q_1 \rfloor = \{|B_1|, \ldots, |B_m|\}$. Q_1 is compliant with Q (written in symbols $Q_1 \propto Q$), iff*

1. *For each $|B_i|$ $(1 \leq i \leq m)$ there exist $k_1, \ldots, k_l$ $(1 \leq k_p \leq n;$ $1 \leq p \leq l)$ such that $|A_{k_1}| \cup \ldots \cup |A_{k_l}| = \bigcup_{p=1}^{l} |A_{k_p}| = |B_i|$.*

2. *For each* $|A_j|$ $(1 \leq j \leq n)$ *there exists* $|B_k|$ $(1 \leq k \leq m)$*, such that* $|A_j| \subset |B_k|$.

As it may be observed—in the case of compliance—we cannot say anything about declarative premises involved in going from a question to question response. The relationship captured by the compliance is a pure subquestionhood relationship. A simple example of question–question response where the reply is compliant to the initiative illustrates this idea.

Example 1 Q is 'Is John coming to the party and can I come?' while Q_1 is 'Is John coming to the party'. Q_1 may be expressed in INQ as $p \vee \neg p$ and Q as $(p \vee \neg p) \wedge (q \vee \neg q)$. $\lfloor Q \rfloor = \{|p \wedge q|, |\neg p \wedge q|, |p \wedge \neg q|, |\neg p \wedge \neg q|\}$ and $\lfloor Q_1 \rfloor = \{|p|, |\neg p|\}$. It is the case that $Q_1 \propto Q$, because both conditions for compliance are met. For the first condition observe that $|p| = |p \wedge q| \cup |p \wedge \neg q|$ and $|\neg p| = |\neg p \wedge q| \cup |\neg p \wedge \neg q|$. For the second condition let us observe that: $|p \wedge q| \subset |p|$; $|p \wedge \neg q| \subset |p|$; $|\neg p \wedge q| \subset |\neg p|$; $|\neg p \wedge \neg q| \subset |\neg p|$.

3 Erotetic implication

3.1 Language $\mathcal{L}_?$

In the following I will use the formal language $\mathcal{L}_?$. $\mathcal{L}_?$ is First-order Logic language enriched with the question-forming operator ? and brackets {, }. Well formed formulae of FoL (defined as usual) are *declarative well-formed formulae* of $L_?$ (d-wffs for short). Expressions of the form $?\{A_1, \ldots, A_n\}$ are *questions* of $L_?$ (e-formulae) provided that $A_1, \ldots, A_n$ are syntactically distinct d-wffs and that $n > 1$. The set $\mathbf{d}Q = \{A_1, \ldots, A_n\}$ is the set of all *direct answers* to the question $Q = ?\{A_1, \ldots, A_n\}$. The question $?\{A_1, \ldots, A_n\}$ might be read as 'Is it the case that A_1 or is it the case that A_2, ..., or is it the case that A_n?'.

For brevity, I will adopt a different notation for one type of questions. So called (*binary*) *conjunctive questions*,[5] namely $?\{A \wedge B, A \wedge \neg B, \neg A \wedge B, \neg A \wedge \neg B\}$ will be written as $? \pm |A, B|$ ('Is it the case that A and is it the case that B?')—cf. (Wiśniewski, 2003, p. 399).

3.2 Pure erotetic implication

A definition of the erotetic implication (e-implication) can be formulated as follows:

[5]For a generalised definition of conjunctive questions see (Urbański, 2001, p. 76).

Definition 6 *A question Q_1 is* e-implied *by a question Q on the basis of a set X of declarative formulae ($Q_1 \rhd_X Q$) iff:*

1. *for each direct answer A to the question Q: $X \cup \{A\}$ entails the disjunction of all the direct answers to the question Q_1, and*
2. *for each direct answer B to the question Q_1 there exists a non-empty proper subset Y of the set of direct answers to the question Q such that $X \cup \{B\}$ entails the disjunction of all the elements of Y.*

Intuitively, erotetic implication ensures the following: (i) if Q is sound[6] and X consists of truths, then Q_1 has a true direct answer as well ('transmission of soundness and truth into soundness'—Wiśniewski, 2003, p. 401), and (ii) each direct answer to Q_1, if true, and if all elements of X are true, it narrows down the class in which a true direct answer to Q can be found ('open-minded cognitive usefulness'—Wiśniewski, 2003, p. 402).

It can be observed that e-implication enables the capturing of the relationship between question-question response on the basis of a set of declarative premises. This is not the case for the compliance (see Definition 5)—the reason for this is that there we are only interested in pure questioning without introducing any information. If X is empty, an e-implication ($Q_1 \rhd Q$) is called *pure* e-implication (Wiśniewski, 2013, p. 76).

Definition 7 $Q_1 \rhd Q$ *iff:*

1. *for each direct answer A to the question Q, A entails the disjunction of all the direct answers to the question Q_1, and*
2. *for each direct answer B to the question Q_1 there exists a non-empty proper subset Y of the set of direct answers to the question Q such that B entails the disjunction of all the elements of Y.*

4 Compliance vs pure e-implication

4.1 Translation

In order to compare presented approaches we need to provide a method of interpretation of formulae having the $\mathcal{Q}$-property in INQ in terms of questions in IEL. I will use the method which was presented by Wiśniewski and Leszczyńska-Jasion (2013).

[6]A question Q is *sound* iff it has a true direct answer (with respect to the underlying semantics).

I will refer to formulae having the $\mathcal{Q}$-property as Q_{INQ} and to questions in IEL as Q_{IEL}. The procedure is as follows:

1. Compute all the possibilities for a given Q_{INQ}.
2. For each possibility choose exactly one wff such that the possibility is just the truth set of the wff.
3. Each such wff is a possible answer for Q_{IEL}.

Let us consider some examples here.

Example 2 Formula in INQ is: $Q_{\text{INQ}} = (p \vee q) \vee \neg(p \vee q)$. Its set of possibilities is the following: $\lfloor Q_{\text{INQ}} \rfloor = \{|p|, |q|, |\neg p| \cap |\neg q|\}$, thus its IEL counterpart might be formulated as follows: $Q_{\text{IEL}} = ?\{p, q, \neg p \wedge \neg q\}$.

Example 3 Formula in INQ is: $Q_{\text{INQ}} = (p \vee \neg p) \wedge (q \vee \neg q)$. Then the set $\lfloor Q_{\text{INQ}} \rfloor = \{|p| \cap |q|, |p| \cap |\neg q|, |\neg p| \cap |q|, |\neg p| \cap |\neg q|\}$ is the set of its possibilities. Thus its IEL counterpart might be formulated as follows: $Q_{\text{IEL}} = ?\{(p \wedge q), (p \wedge \neg q), (\neg p \wedge q), (\neg p \wedge \neg q)\}$ (or using abbreviation according to mentioned convention: $Q_{\text{IEL}} = ? \pm |p, q|$).

In the following I will use questions in the IEL notation.

4.2 Examples of question responses

Let us now consider some examples of question responses in the light of pure e-implication and compliance.

Example 4 Let us consider the following case. Initiative: $? \pm |p, q|$ and response: $?\{p, \neg p\}$:

- $?\{p, \neg p\} \propto ? \pm |p, q|$
- $?\{p, \neg p\} \rhd ? \pm |p, q|$

Example 5 Let us take the following initiative: $?\{p \wedge q, p \wedge \neg q, \neg p\}$ and response: $?\{p, \neg p\}$:

- $?\{p, \neg p\} \propto ?\{p \wedge q, p \wedge \neg q, \neg p\}$
- $?\{p, \neg p\} \rhd ?\{p \wedge q, p \wedge \neg q, \neg p\}$

Example 6 E-implication and compliance hold also for the following initiative: $?\{p, q, \neg p \wedge \neg q\}$ and response: $?\{p \vee q, \neg p \wedge \neg q\}$:

- $?\{p \vee q, \neg p \wedge \neg q\} \;\propto\; ?\{p, q, \neg p \wedge \neg q\}$
- $?\{p \vee q, \neg p \wedge \neg q\} \rhd ?\{p, q, \neg p \wedge \neg q\}$

Example 7 And also for the following initiative: $?\{\neg p, p \wedge q, p \wedge \neg q\}$ and response: $?\{p \rightarrow q, p \wedge \neg q\}$:

- $?\{p \rightarrow q, p \wedge \neg q\} \;\propto\; ?\{\neg p, p \wedge q, p \wedge \neg q\}$
- $?\{p \rightarrow q, p \wedge \neg q\} \rhd ?\{\neg p, p \wedge q, p \wedge \neg q\}$

There are, however, cases where e-implication holds, while the response might not be treated as a compliant one.

Example 8 For the following initiative: $?\{p, \neg p, q, \neg q\}$ and response: $?\{p, \neg p\}$ we have:

- $?\{p, \neg p\} \not\propto ?\{p, \neg p, q, \neg q\}$
- $?\{p, \neg p\} \rhd ?\{p, \neg p, q, \neg q\}$

Example 9 Similarly for $?\{p, q, \neg p \wedge \neg q\}$ as the initiative, and $?\{p, \neg p\}$ as the response:

- $?\{p, \neg p\} \not\propto ?\{p, q, \neg p \wedge \neg q\}$
- $?\{p, \neg p\} \rhd ?\{p, q, \neg p \wedge \neg q\}$

Example 10 Both—compliance and e-implication—do not hold in the case when we want the question response to be much more detailed than the initiative, as in the following case.

- $?\{p, \neg p, q, \neg q\} \not\propto ?\{p, \neg p\}$
- $?\{p, \neg p, q, \neg q\} \;\not\rhd ?\{p, \neg p\}$

4.3 Compliance is stronger than pure e-implication

Theorem 1 *If* $Q_1 \propto Q$ *then* $Q_1 \rhd Q$*.*

Proof. Suppose that $Q_1 \propto Q$. We should show that both conditions of e-implication for $Q_1 \rhd Q$ are met (cf. Definition 7).

The first condition for e-implication is met for obvious reasons: as only classical tautologies have $\mathcal{Q}$-property in INQ, Q_1 must be a safe question and thus a sound one (see Wiśniewski, 2013, p. 77, Corollary 7.22).

Now, let us consider the second condition for e-implication, which states that for each direct answer B to the question Q_1 there exists a proper non-empty subset Y of the set of direct answers to the question Q such that B entails the disjunction of all the elements of Y. Compliance demands that for each answer B to the question Q_1 there exists a non-empty subset Y of the set of direct answers to question Q such that the truth set for B is equivalent to the truth set for the disjunction of all formulae in Y. In other words Y is such that B entails the disjunction of all the elements of Y and the disjunction of all the elements in Y entails B. When the stronger condition for compliance will hold also the condition for e-implication will be satisfied. □

We may observe this asymmetry between compliance and e-implication in Example 9. The reason why $?\{p, \neg p\} \not\propto ?\{p, q, \neg p \wedge \neg q\}$ is that there exists the answer to Q_1—namely $\neg p$—for which one cannot point the subset Y, which will meet the first condition for compliance. At the same time e-implication holds because the second condition for e-implication is fulfilled.

When we take a closer look on the Example 8 we will also notice that the second condition of compliance definition (called the restriction clause) makes it stronger than pure e-implication. The intuition behind this clause is that—while proposing Q_1—we cannot rule out a possible answer without providing any information. We may say that the level of information while passing form Q to Q_1 remains the same—the information needed to answer the Q is always enough to answer the Q_1 (cf. Cornelisse, 2009, p. 12). For e-implication it is enough that for each direct answer A to question Q, A entails disjunction of all answers to Q_1. For compliance for each A (which is answer to question Q) there should exist an answer B to question Q_1 such that A entails B. If we consider Example 8 this condition for compliance is not met for the following answers to Q: q and $\neg q$.

5 Summary

In this paper I have introduced approaches to question dependency based in IEL and INQ framework. It is possible to compare compliance and e-implication despite many differences in the background frameworks for these concepts. The obtained result is that compliance is a stronger relationship than pure e-implication. This is the consequence of an intuition behind compliance, saying that a compliant question response should not rule out any possibilities from the initiative. As such compliance might serve as a good source of inspiration for strengthening e-implication.

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